Q1a

a) $(x^2-y^2)(x^2-y^2)$

$$\boxed{x^4-2x^2y^2+y^4}$$

Q1b

b) $(2x-3y+3)(2x+3y-3)(x-y)$

$4x^2+\cancel{6xy}-\cancel{6x}$

$-\cancel{6xy}-9y^2+9y$

$+\cancel{6x}+9y-9$

$(4x^2-9y^2+18y-9)(x-y)$

$$\boxed{\begin{aligned}&4x^3-9xy^2+18xy-9x\\&\quad-4x^2y+9y^3-18y^2+9y\\&\qquad\qquad\text{UNITS}^3\end{aligned}}$$

Q2

$$(ax)(2x)(x) = 8x^3$$
$$a2\cancel{x^3} = 8\cancel{x^3}$$
$$2a = 8 \qquad a = 4$$
$$(by)(y)(-3y) = -9y^3$$
$$-3b\cancel{y^3} = -9\cancel{y^3}$$
$$-3b = -9 \qquad b = 3$$
$$(4x+3y)(2x+y)(x-3y)$$
$$(8x^2 + \underbrace{4xy + 6xy}_{} + 3y^2)(x-3y)$$
$$(8x^2 + 10xy + 3y^2)(x-3y)$$

$$x(8x^2+10xy+3y^2)$$
$$-3y(8x^2+10xy+3y^2)$$
$$8x^3 + 10x^2y + 3xy^2$$
$$-24x^2y - 30xy^2 - 9y^3$$
$$8x^3 - 14x^2y - 27xy^2 - 9y^3$$

$$\boxed{a=4 \quad b=3 \quad c=-14 \quad d=-27}$$

Q3

$$xy\underline{(x^4 - y^4)}$$

DIFFERENCE OF TWO SQUARES

$$xy(x^2 + y^2)\underline{(x^2 -}$$

DIFFERENCE OF TWO SQUARES.

$$\boxed{xy(x^2 + y^2)(x + y)(x}$$

---

Q4

BY INSPECTION

$+2x^3$

$-2x^3$

$$(2x - 1)(2x^3 + \quad x^2 - 18x - 9)$$

$-x^2$

$-37x^2$

LONG DIVISION

$$
\begin{array}{r l}
 & 2x^3 + x^2 - 18x - 9 \\
\hline
2x-1 \,\big) & \cancel{4x^4} - 37x^2 + 9 \\
 & -(\cancel{4x^4} - 2x^3) \\
\hline
 & +\cancel{2x^3} - 37x^2 + 9 \\
 & -(+\cancel{2x^3} - x^2) \\
\hline
 & -\cancel{36x^2} + 9 \\
 & -(-\cancel{36x^2} + 18x) \\
\hline
 & \cancel{-18x + 9} \\
 & -(\cancel{-18x + 9}) \\
\hline
 & 0
\end{array}
$$

$$\boxed{2x^3 + x^2 - 18x - 9}$$

Q5a

a) $6x^4+7x^3-27x^2-28x+12$

BY INSPECTION

$+8x$

$-2x^3$

$9x^3$

$$(2x+3)(3x^3-x^2-12x+4)$$

$-3x^2$

$-24x^2$

$-36x$

NO REMAINDER

REMAINDER $=0$

Q5b

b) $(2x+3)$ IS A FACTOR FROM PART (a)

$$(2x+3)(3x^3-x^2-12x+4)$$

BY INSPECTION $(x+2)(3x^2-7x+2)$

FACTORISE QUADRATIC

$$(2x+3)(x+2)(3x^2-7x+2)$$

6

6

$-6$ $-1$

$$(3x-6)(3x-1)$$

$\div 3$

$$(x-2)(3x-1)$$

$$\boxed{(2x+3)(x+2)(x-2)(3x-1)}$$

Q6a

a) $3x^4+x^3-12x^2-49x-15$

$+x^3$ $-12x^2$

$$(3x+1)(x^3-4x-15)$$

$-4x$ $-45x$

$$\boxed{(3x+1)(x^3-4x-15)}$$

Q6b

b) $(3x+1)(x^3-4x-15)$

BY INSPECTION

$$(x-3)(x^2+3x+5)$$

$+5x$ $+3x^2$ $-3x^2$ $-9x$

↳ CANT BE FACTORISED

SO FULLY FACTORISED

$$\boxed{(3x+1)(x-3)(x^2+3x+5)}$$

Q6c

c)

$$(3x+1)(x-3)(x^2+3x+5)=0$$

$x=-\frac{1}{3}$ $\quad x=3$

DISCRIMINANT

$b^2-4ac<0$

$x^2+3x+5$

$a=1 \quad b=3 \quad c=5$

$(3)^2-4(1)(5)<0$

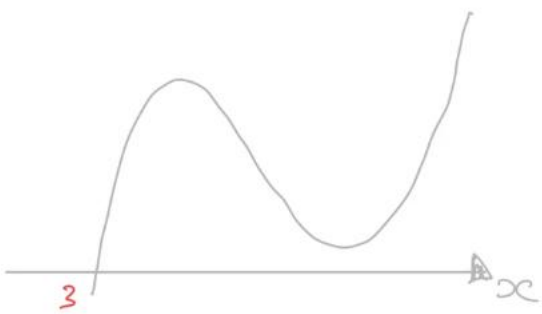
$9-20<0$

$-11<0$ $\therefore$ NO REAL ROOTS

EXACTLY 2 REAL ROOTS

$x=-\frac{1}{3}$ AND $x=3$

Q7



IF $f(3)=0$ THEN $(x-3)$ IS A FACTOR

$$2x^3-x^2-11x-12$$

$$-6x^2 \quad -15x$$

$$(x-3)(2x^2+5x+4)$$

$$+5x^2 \quad +4x$$

DISCRIMINANT

$$2x^2+5x+4 \qquad b^2-4ac<0$$

$a=2 \quad b=5 \quad c=4$

$$(5)^2-4(2)(4)<0$$

$$25-32<0$$

$$-7<0 \therefore \text{NO REAL ROOTS}$$

$$\boxed{\text{QUADRATIC PART HAS NO REAL ROOTS} \quad \text{ONLY ROOT} = 3}$$

Q8a

a)

$f(-1)$

$$2(-1)^4-15(-1)^3-10(-1)^2+105(-1)+98$$
$$=0$$

$f(-2)$

$$2(-2)^4-15(-2)^3-10(-2)^2+105(-2)+98$$
$$=0$$

$$\boxed{f(-1)=0 \qquad f(-2)=0}$$

Q8b

b)

IF $f(-1)=0$ AND $f(-2)=0$

$(x+1)$ AND $(x+2)$ ARE FACTORS

DIVIDE BY $(x+1)$

$$(x+1)(2x^3-17x^2+7x+98)$$

DIVIDE BY $(x+2)$

$$(x+1)(x+2)(2x^2-21x+49)$$

FACTORISE QUADRATIC

$$(x+1)(x+2)(2x^2-21x+49) \quad 98$$

$$(2x-14)(2x-7) \quad \begin{matrix} -2 & -49 \\ -14 & -7 \end{matrix}$$

$\div 2$

$$(x+1)(x+2)(x-7)(2x-7)$$

$f(x)=0$

$$(x+1)(x+2)(x-7)(2x-7)=0$$

$$\boxed{x=-1 \quad x=-2 \quad x=7 \quad x=\frac{7}{2}}$$

Q9

"Extended Factor Theorem"

If $(ax-b)$ is a factor of $f(x)$, then $f\left(\frac{b}{a}\right)=0$

By the Factor Theorem, if $(2x-5)$ is a factor, then

$$f\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^3 + k\left(\frac{5}{2}\right)^2 - 11\left(\frac{5}{2}\right) - 60 = 0$$

So $\frac{125}{4} + \frac{25k}{4} - \frac{55}{2} - 60 = 0$

$$\frac{125 + 25k - 110 - 240}{4} = 0$$

$$25k - 225 = 0 \Rightarrow \boxed{k = 9}$$

$2x^3 + 9x^2 - 11x - 60 = (2x-5)(x^2 + 7x + 12)$ by inspection

$$\boxed{= (2x-5)(x+3)(x+4)}$$

Q10

[5]

"Extended Factor Theorem"

If $f\left(\frac{b}{a}\right)=0$, then $(ax-b)$ is a factor of $f(x)$

If we let $y=x^2$, then

$9x^4-40x^2+16=9y^2-40y+16$

and $(9x^2-4)=(9y-4)$

When $y=4/9$,

$9(4/9)^2-40(4/9)+16=0$

shows that $(9y-4)$ is a factor of $9y^2-40y+16$.

Here we just do everything in terms of $x^2$, instead of making the explicit substitution.

$9x^4-40x^2+16=9(x^2)^2-40(x^2)+16$

If $x^2=4/9$,

$9(4/9)^2-40(4/9)+16=\frac{16}{9}-\frac{160}{9}+\frac{144}{9}=0$

So $(9x^2-4)$ is a factor of $9x^4-40x^2+16$, by the Factor Theorem.

$9x^4-40x^2+16=(9x^2-4)(x^2-4)=0$

$(3x+2)(3x-2)(x+2)(x-2)=0$

The solutions are

$x=-2/3,\ 2/3,\ -2,\ 2$

Q11a

"Extended Factor Theorem"

If $f\left(\frac{b}{a}\right)=0$, then $(ax-b)$ is a factor of $f(x)$

If $a=0$, $\frac{2}{a}$ is undefined. But when $a=0$,

$(ax-2)=-2$ and $3ax^2+(a-6)x-2=-6x-2$

a)

If $a=0$, $-2$ is a factor of

$-6x-2=-2(3x+1)$.

If $a\neq0$,

$3a\left(\frac{2}{a}\right)^2+(a-6)\left(\frac{2}{a}\right)-2$

$=\frac{12}{a}+2-\frac{12}{a}-2=0$

So by the Factor Theorem,

$(ax-2)$ is a factor of

$3ax^2+(a-6)x-2$.

Combining these results, $(ax-2)$ is a factor of $3ax^2+(a-6)x-2$ for any value of $a$.

Q11b

"Extended Factor Theorem"

If $f\left(\frac{b}{a}\right)=0$, then $(ax-b)$ is a factor of $f(x)$

b) By the Factor Theorem, if $-\frac{1}{a-4}$ is a root, then $((a-4)x+1)$ is a factor.

So

$$3ax^2+(a-6)x-2=(ax-2)((a-4)x+1)$$
$$=(a^2-4a)x^2+(8-a)x-2$$

Equating the $x^2$ coefficients,

$$3a=a^2-4a \Rightarrow a^2-7a=a(a-7)=0 \Rightarrow a=0 \text{ or } 7$$

Equating the $x$ coefficients,

$$a-6=8-a \Rightarrow 2a=14 \Rightarrow a=7$$

The correct value of $a$ must satisfy both equations!

$$\boxed{\text{Therefore } a=7}$$

Q12a

a) By the Remainder Theorem, the remainder will be

$$8\left(-\tfrac{1}{2}\right)^3+6\left(-\tfrac{1}{2}\right)^2-\left(-\tfrac{1}{2}\right)-2$$
$$=-1+\frac{3}{2}+\frac{1}{2}-2=\boxed{-1}$$

Q12b

**Method 1**

By the Remainder Theorem, the remainder will be

$$6\left(-\tfrac{1}{2}\right)^2-\left(-\tfrac{1}{2}\right)-2=\tfrac{3}{2}+\tfrac{1}{2}-2=\boxed{0}$$

**Method 2**

$6x^2-x-2=(2x+1)(3x-2)$, so $(2x+1)$ is a factor of $6x^2-x-2$.

$\boxed{\text{The remainder will be } 0}$

**Method 3**

$$6x^2-x-2=(8x^3+6x^2-x-2)-8x^3$$

So by the Remainder Theorem, the remainder will be

$$-1-8\left(-\tfrac{1}{2}\right)^3=-1+1=\boxed{0}$$

From part (a), $8\left(-\tfrac{1}{2}\right)^3+6\left(-\tfrac{1}{2}\right)^2-\left(-\tfrac{1}{2}\right)-2=-1$

**Method 4**

(Use algebraic division)

Q13

dividend

remainder

$$\frac{f(x)}{(x+4)}=g(x)+\frac{c}{(x+4)}$$

divisor

quotient

divisor

So $f(x)=g(x)(x+4)+c$

$$2x^3+(a+b)x^2+(a-b)x-3=\underbrace{\left(2x^2+(2a+3)x+(2b-5)\right)}_{\text{quotient}}(x+4)+\underbrace{c}_{\text{remainder}}$$

$$=2x^3+(2a+11)x^2+(8a+2b+7)x+(8b+c-20)$$

Comparing coefficients,

$a+b=2a+11 \Rightarrow a-b=-11$ ① $x^2$ terms

$a-b=8a+2b+7 \Rightarrow 7a+3b=-7$ ② $x$ terms

$8b+c-20=-3 \Rightarrow 8b+c=17$ ③ constant terms

$3\times$①+②: $10a=-40 \Rightarrow a=-4 \Rightarrow b=7$

Substitute into ③: $8(7)+c=17 \Rightarrow c=-39$

$$\boxed{\begin{array}{l}\text{The solution is}\\ a=-4 \quad b=7 \quad c=-39\end{array}}$$